A-level
MATHEMATICS
7357/1
Paper 1

Mark scheme

June 2022

Version: 1.1 Final Mark Scheme

226A7357/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses . If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

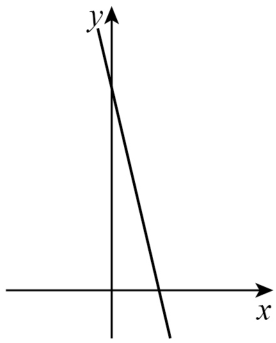
Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles the correct answer	1.2	B1	$x^2 + y^2 = 1$
	Question 1 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles the correct answer	1.1b	B1	2
	Question 2 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles the correct answer	2.2a	R1	$y = 2\log_4 x$
	Question 3 Total		**1**	

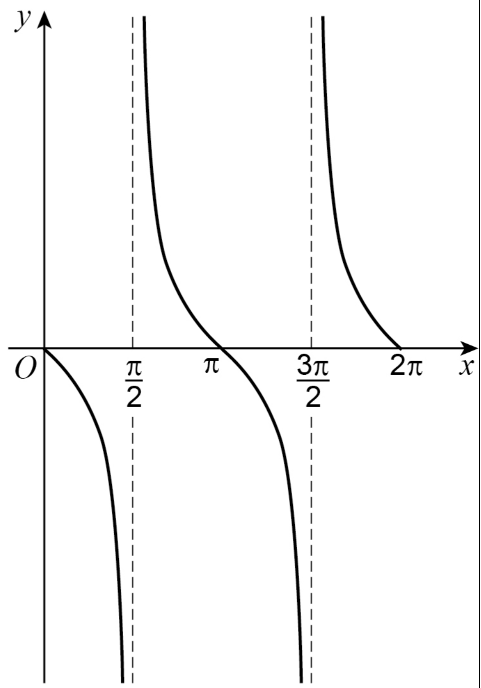
Q	Marking instructions	AO	Marks	Typical solution
4	Ticks the correct box	2.2a	R1	
	Question 4 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5	Differentiates to obtain a correct derivative either $4(x-2)^3$ OE or $4x^3-24x^2+48x-32$ PI by -32 obtained with no errors seen in evaluating $\frac{dy}{dx}$	1.1b	B1	$\frac{dy}{dx}=4(x-2)^3$ When x = 0 $\frac{dy}{dx}=-32$ $y=16$ $y=-32x+16$
	Substitutes x = 0 into their $\frac{dy}{dx}$ to obtain a numerical value or PI by constant from their $\frac{dy}{dx}$ or PI by -32 obtained with no errors seen in evaluating $\frac{dy}{dx}$	1.1a	M1	
	Obtains $y=-32x+16$ ACF Award the mark at the first opportunity and ISW any incorrect rearrangement No errors seen	1.1b	A1	
	Question 5 Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Expands to obtain the first two terms Can be unsimplified Condone sign error	1.1a	M1	$\left(1-\frac{x}{2}\right)^{\frac{1}{2}} \approx 1+\left(\frac{1}{2}\right)\left(-\frac{x}{2}\right)$ $\approx 1-\frac{1}{4}x$
	Obtains $1-\frac{1}{4}x$ OE Accept if listed as two separate terms. Ignore any extra terms	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	States or uses at least one small angle approximation correctly either $\sin kx \approx kx$ or $\sqrt{\cos x} \approx \sqrt{1-\frac{x^2}{2}}$	3.1a	M1	$\sin(4x)+\sqrt{\cos x} \approx 4x+\sqrt{1-\frac{x^2}{2}}$ $\approx 4x+\left(1-\frac{x^2}{4}\right)$ $\approx 1+4x-\frac{1}{4}x^2$
	Uses both small angle approximations correctly for sine and cosine $\sin kx \approx kx$ and $\sqrt{\cos x} \approx \sqrt{1-\frac{x^2}{2}}$ Must have eliminated all trig expressions Inconsistent variables for angles must eventually be consistent to be awarded A1	1.1b	A1	
	Uses their expansion from **(a)** Must have replaced x with x^2 or Applies binomial theorem correctly to $\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$ ignore any extra terms	3.1a	M1	
	Completes argument to obtain $4x+\left(1-\frac{x^2}{4}\right)$ or $1+4x-\frac{1}{4}x^2$ Accept any order of terms Ignore higher powers of x Must be in terms of x Do not ISW	2.1	R1	
	Subtotal		**4**	

	Question 6 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
7	Sketches one of the sections of the curve shown below Either  or  Condone translations Do not allow the end points of the curve turning to intersect the asymptote	1.2	B1	
	Sketches the three branches within the interval from 0 to 2π Condone overlapping branches Asymptotes need not be drawn	1.1a	M1	
	Completes fully correct sketch with asymptotes drawn at approximately the correct positions Labelling not required and can be ignored Ignore anything after 2π or to the left of O	1.1b	A1	
	Question 7 Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(i)	Obtains correct y-intercept at P (0,5) or y = 5 seen anywhere PI by correct equation for line PQ	3.1a	B1	At P $x = 0 \Rightarrow y = 5$ Line PQ $3x - 5y = -25$ $5x + 3y = 83$ (10, 11)
	Obtains equation of PQ with correct gradient. For example $y = \frac{3}{5}x + c$ or $5y - 3x = k$ or Forms an equation for the distance or distance squared from (0,5) to a point on L_2 For example, $d^2 = x^2 + \left(-\frac{5}{3}x + \frac{68}{3}\right)^2$	3.1a	M1	
	Obtains correct equation ACF	1.1b	A1	
	Solves simultaneous equations for their PQ and L_2 to obtain values for x and y Their PQ must not be a horizontal or vertical line Condone errors in rearrangement of the equation(s) or Minimises their distance or distance squared equation to find one coordinate	3.1a	M1	
	Obtains (10, 11) or $x = 10,\ y = 11$	1.1b	A1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(ii)	Uses the distance formula to find the value of PQ or PQ^2 or Uses Pythagoras theorem with 10^2+6^2 seen If the coordinates of P and Q are **incorrect**, differences in x and y must be clearly shown for M1	1.1a	M1	$PQ^2=(10-0)^2+(11-5)^2$ $PQ=\sqrt{136}=2\sqrt{34}$
	Completes demonstration to show that k = 2 Must have shown clear use of distance formula Condone not seeing x = 0 substituted in the distance formula Answer of $2\sqrt{34}$ and no working shown scores M1 R0	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)(i)	Uses a valid method to find a. Evidence could be: Forming the equation of the line mid-way between L_1 and L_2 $5x+3y=49$ seen or Using (a , -17) as the mid-point of a line segment from L_1 to L_2 For example: $5x+3(-17)=15 \quad x=13.2$ $5x+3(-17)=83 \quad x=26.8$ $a=\frac{(26.8+13.2)}{2}=20$ or Finding the mid-point of PQ their (5 , 8) and using the gradient of L_1 and $L_2 = -\frac{5}{3}$ For example: $8+5(-5)=-17$ $a=5+5(3)=20$ or Substitutes y = 17 and x = a into $y=\frac{3}{5}x+5$	3.1a	M1	$5x+3y=49$ $5a+3(-17)=49$ $a=20$
	Deduces a = 20	2.2a	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)(ii)	Forms expression of the form $(x \pm a)^2 + (y \pm 17)^2$ using a or their value of a	1.1a	M1	$(x-20)^2+(y+17)^2=34$
	Obtains correct equation for their value of a and their radius^2 = $\frac{17k^2}{2}$ from part (a)(ii) for an integer value of k Condone $\left(\sqrt{34}\right)^2$	1.1b	A1F	
	Subtotal		**2**	

	Question 8 Total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Forms an appropriate equation in x only by either using the differences of at least one pair of terms or Using the mean of the first and third term = the second term Condone missing brackets or Forms two simultaneous equations in x and d or Substitutes x = 5 and demonstrates that the three terms obtained, 15, 26 and 37 have a common difference of 11 or Shows that the sum formula for an arithmetic series works when x = 5 The approaches that substitute x = 5 score a maximum of M1 A0 R0	3.1a	M1	$5x+1-(2x+5)=6x+7-(5x+1)$ $3x-4=x+6$ $x=5$ Therefore $x=5$ is the only solution
	Obtains a correct equation or Obtains two correct simultaneous equations in x and d Need not be simplified	1.1b	A1	
	Solves to conclude that $x=5$ is the only solution Must include the word 'only' OE	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(i)	Obtains 15	1.1b	B1	$a=15$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(ii)	Obtains 11	1.1b	B1	$d=11$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	Forms an expression for the sum to N or N +1 terms using their a and d values Need not be simplified Condone missing brackets or use of n or Uses a trial and improvement method obtaining sums for two different values of n	3.1a	M1	$S_N = \frac{N}{2}\left(2\times 15 + 11(N-1)\right)$ $\frac{N}{2}\left(2\times 15 + 11(N-1)\right) = 100000$ $N = 133.9...$ $N = 133$
	Forms an equation or inequality using their expression and $100000 \pm k$ where $0 \le k \le 11$ or Uses trial and improvement to obtain one sum below 100000 and one sum above 100000 for consecutive integers	1.1a	M1	
	Obtains either 133.9.. or 132.9.. or $N > 132$ or $N < 134$ or Obtains the sum of 98553 when n = 133 and obtains the sum of 100031 when n = 134	1.1b	A1	
	Obtains 133 having solved a correct quadratic This mark can be recovered if N = 133 and N = 134 are correctly checked	3.2a	A1	
	Subtotal		**4**	

	Question 9 Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Recalls or uses the area of sector = $\frac{1}{2}r^2\theta$ r can be any letter or OA or OB or any consistent value throughout	1.2	B1	Area of sector = $\frac{1}{2}r^2\theta$ Area of triangle = $\frac{1}{2}ab\sin C$ Hence $\frac{1}{2}ab\sin C = \left(\frac{1}{2}\right)\frac{1}{2}r^2\theta$ $\frac{1}{2}r^2\sin\theta\cos\theta = \frac{1}{2}\left(\frac{1}{2}r^2\theta\right)$ $2\sin\theta\cos\theta = \theta$ $\theta = \sin 2\theta$
	Forms an equation relating the area of the triangle OAC and sector using $\frac{1}{2}bh = k\frac{1}{2}r^2\theta$ where $k > 0$	3.1a	M1	
	Deduces area of triangle is $\frac{1}{2}r\cos\theta \times r\sin\theta$ OE Must use trigonometry for height and base	2.2a	B1	
	Completes reasoned argument with clear use of double angle identity to show that $\theta = \sin 2\theta$ or $\sin 2\theta = \theta$	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Rearranges to obtain $\theta - \sin 2\theta = 0$ or $\sin 2\theta - \theta = 0$ (which may be seen in conclusion) **and** evaluates $\theta - \sin 2\theta$ or $\sin 2\theta - \theta$ at $\frac{\pi}{5}$ (0.6284) and $\frac{2\pi}{5}$ (1.257) Evaluates using any two other appropriate values inside the interval but either side of root.	1.1a	M1	$\theta = \sin 2\theta \Rightarrow \theta - \sin 2\theta = 0$ Let $f(\theta) = \theta - \sin 2\theta$ $f\left(\frac{\pi}{5}\right) = -0.3227... < 0$ $f\left(\frac{2\pi}{5}\right) = 0.6688... > 0$ Hence solution lies between $\frac{\pi}{5}$ and $\frac{2\pi}{5}$
	Completes reasoned argument with reference to change of sign and evidence of correct evaluation accepting values rounded or truncated to 1 sf Must refer to $\frac{\pi}{5}$ and $\frac{2\pi}{5}$ in the conclusion	2.1	R1	
	Subtotal		**2**	

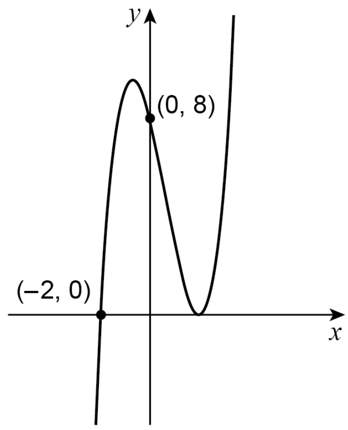
Q	Marking instructions	AO	Marks	Typical solution
10(c)(i)	Differentiates $\sin 2\theta$ to obtain $2\cos 2\theta$ OE PI by correct θ_2 or θ_3 PI by sight of $2\cos\frac{2\pi}{5}$	1.1b	B1	$f(\theta) = \theta - \sin 2\theta$ $f'(\theta) = 1 - 2\cos 2\theta$ $\theta_{n+1} = \theta_n - \frac{\theta_n - \sin 2\theta_n}{1 - 2\cos 2\theta_n}$ $\theta_2 = 1.4732575...$ $\theta_3 = 1.0413241...$ $\theta_3 = 1.041$
	Obtains a correct expression for $\theta_n - \frac{\theta_n - \sin 2\theta_n}{1 - 2\cos 2\theta_n}$ Accept use of ANS or $\frac{\pi}{5}$ Condone missing or incorrect subscript PI by correct θ_2 or θ_3 AWRT θ_2 1.473	1.1a	M1	
	Obtains correct θ_3 AWRT θ_3 1.041	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)(ii)	Explains that more iterations could be used Accept keep on using Newton Raphson, keep re-iterating	2.4	E1	Use more iterations
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)(iii)	States that $\mathrm{f}'\left(\frac{\pi}{6}\right)=0$	2.4	E1	$\mathrm{f}'\left(\frac{\pi}{6}\right)=0$ The value is on a stationary point
	Explains a general reason for the Newton Raphson iteration not to converge to a particular root Accept only • too close to a stationary point • the value is on a stationary point • the tangent does not cross the x-axis • it converges to a different root • the formula is undefined Accept equivalents to these five bullet points only	2.4	E1	
	Subtotal		**2**	

	Question 10 Total		**12**	

Q	Marking instructions	AO	Marks	Typical solution
11(a)	Substitutes $x = -2$ into $\mathrm{p}(x)$ Condone missing brackets	1.1a	M1	$\mathrm{p}(-2) = (-2)^3 + (b+2)(-2)^2 + 2(b+2)(-2) + 8$ $= -8 + 4b + 8 - 4b - 8 + 8$ $= 0$ Hence $(x+2)$ is a factor of $\mathrm{p}(x)$ for all values of b
	Demonstrates clearly that $\mathrm{p}(-2) = 0$ Must see numerical evaluation of powers of -2 and either $-8 + 4b + 8 - 4b - 8 + 8 = 0$ or $-8 + 4(b+2) - 4(b+2) + 8 = 0$ or $-8 + (b+2)(4-4) + 8 = 0$	2.1	A1	
	Concludes and states that $(x+2)$ is a factor for **all/any values of *b***	2.4	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)(i)	Sketches cubic graph with correct orientation and two turning points	1.2	B1	
	Sketches any cubic that would only ever meet the x-axis at exactly two points	2.2a	M1	
	Sketches a correctly orientated cubic graph that has a • single root labelled x = -2 • y intercept labelled at 8 • repeated root on the positive x-axis Ignore any value shown at the other root	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)(ii)	Deduces a pair of possible factors of (x^2+bx+4) Either $(x+2)^2$ or $(x-2)^2$ or $(x+1)(x+4)$ or $(x-1)(x-4)$ or Uses b^2-4ac OE PI $b=4$ or $b=-4$ or b^2-16 seen	2.2a	M1	
	Identifies the quadratic factor as $(x-2)^2$ or Obtains $b^2-16=0$ OE PI by $b=\pm4$ or $b=-4$	2.1	R1	$b^2-4ac=0$ $b^2-16=0$ $\Rightarrow b=\pm4$ $b=4$ gives only one point of intersection
	Obtains either $b=\pm4$ or $b=-4$	1.1a	M1	$\therefore b=-4$
	Rejects $b=4$ giving a reason and concludes $b=-4$ Valid reasons would be: • only one factor or root • only one point of intersection with x-axis • it would be $(x+2)^3$ which only has one point of intersection	2.4	R1	
	Subtotal		**4**	

	Question 11 Total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
12(a)(i)	Uses $\dfrac{a}{1-r}$	1.1a	M1	$S_\infty = \dfrac{1}{1-\frac{1}{2}} = 2$
	Obtains 2	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(a)(ii)	Deduces $a = \dfrac{1}{2}$ and $r = \dfrac{1}{2}$ or Deduces $\sum_{n=1}^{\infty}\left(\sin 30^\circ\right)^n$ = their part **(a)(i)** -1 or Deduces the answer is half of their answer in part **(a)(i)**	2.2a	M1	$\sum_{n=1}^{\infty}\left(\sin 30^\circ\right)^n = \dfrac{1}{2}+\dfrac{1}{4}+...$ $= \dfrac{\frac{1}{2}}{1-\frac{1}{2}}$ $= 1$
	Obtains 1	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)	Forms equation $\frac{a}{1-r}=2-\sqrt{2}$ If the above is not seen then condone $\frac{\cos\theta}{1-\cos\theta}=2-\sqrt{2}$ or Condone use of a numerical value for a where a > 0	3.1a	M1	$\sum_{n=0}^{\infty}(\cos\theta)^n=2-\sqrt{2}$ $\frac{a}{1-r}=2-\sqrt{2}$
	Uses $a=1$ and $r=\cos\theta$	1.1b	B1	$a=1$ $r=\cos\theta$
	Obtains either $r=1-\frac{1}{2-\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$ or $\cos\theta=1-\frac{1}{2-\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$ ACF	1.1b	A1	$1-\cos\theta=\frac{1}{2-\sqrt{2}}$ $\cos\theta=1-\frac{1}{2-\sqrt{2}}$ $\theta=\frac{3\pi}{4}$
	Deduces $\theta=\frac{3\pi}{4}$	2.2a	R1	
	Subtotal		**4**	

	Question 12 Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Substitutes y = 0 and x = 16 correctly into $x^2+y^2=a\sqrt{x}-y$	3.4	M1	$x^2+y^2=a\sqrt{x}-y$ $16^2+0^2=a\sqrt{16}-0$ $256=4a$ $a=64$
	Obtains a = 64	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Differentiates implicitly with either $2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or $-\dfrac{\mathrm{d}y}{\mathrm{d}x}$ seen	3.1b	B1	$2x+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{64}{2}x^{-\frac{1}{2}}-\dfrac{\mathrm{d}y}{\mathrm{d}x}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 2x=\dfrac{32}{\sqrt{x}}$ $x^{\frac{3}{2}}=16$ $x=6.3496...$ $(6.3496...)^2+y^2=64\sqrt{6.3496...}-y$ $y=10.51$ Maximum height is approximately 10.5 metres
	Differentiates any two of the four terms correctly. Can be in terms of a or with their a value	1.1a	M1	
	Obtains a fully correct differentiated equation Can be in terms of a Follow through their a value $2x+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{a}{2}x^{-\frac{1}{2}}-\dfrac{\mathrm{d}y}{\mathrm{d}x}$	1.1b	A1F	
	Uses $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	1.1a	M1	
	Substitutes their numerical x value where 0 < x < 16, into the model with their a value	3.4	M1	
	Obtains a value for y AWRT 10.51 and concludes that the maximum height is approximately 10.5 metres AG Condone equals Must state units CSO	3.2a	R1	
	Subtotal		**6**	

Q	Marking instructions	AO	Marks	Typical solution
13(c)	States or infers that the entrance is unlikely to be a smooth curve Accept: • The cave has dents • Entrance is not perfectly smooth Ignore comments about the floor or the vertical cross section	3.5b	E1	The entrance to the cave is unlikely to be perfectly smooth
	Subtotal		**1**	

	Question 13 Total		**9**	

<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td rowspan="3">14(a)</td><td>Finds positive or negative y-values for 5 x-values with h = 0.75
PI by
AWRT 5.28 or AWRT – 5.28
or
Uses 6 x-values and obtains AWRT 5.42 or AWRT – 5.42
In this case maximum mark is M1A0 A0</td><td>1.1a</td><td>M1</td><td rowspan="3"><table><tr><th>x_n</th><th>y_n</th></tr><tr><td>1</td><td>0</td></tr><tr><td>1.75</td><td>-2.51827</td></tr><tr><td>2.5</td><td>-2.74887</td></tr><tr><td>3.25</td><td>-1.76798</td></tr><tr><td>4</td><td>0</td></tr></table>
$\frac{0.75}{2}\left(0+0+2\left(-2.51827-2.74887-1.76798\right)\right)$

Area $\approx$ 5.28</td></tr>
<tr><td>Uses the trapezium rule correctly with h = 0.75 and correct y-values
Accept rounded or truncated values to 3 significant figures.</td><td>1.1b</td><td>A1</td></tr>
<tr><td>Obtains AWRT 5.28
Condone AWRT – 5.28</td><td>3.2a</td><td>A1</td></tr>
<tr><td></td><td align="right">Subtotal</td><td></td><td>3</td><td></td></tr>
</table>

Q	Marking instructions	AO	Marks	Typical solution
14(b)	Sets up integration by parts Condone u and v' in wrong order Must have expressions for u , u' , v and v' with evidence of some integration	3.1a	M1	$\int_1^4 (2x-8)\ln x \,\mathrm{d}x$ $u=\ln x \quad u'=\frac{1}{x}$ $v'=2x-8 \quad v=x^2-8x$ $(x^2-8x)\ln x-\int x-8 \,\mathrm{d}x$ $=\left[(x^2-8x)\ln x-\frac{x^2}{2}+8x\right]_1^4$ $=\left((16-32)\ln 4-\frac{4^2}{2}+32\right)-\left((1-8)\ln 1-\frac{1^2}{2}+8\right)$ $=-16\ln 2^2+24-\frac{15}{2}$ $=\frac{33}{2}-32\ln 2$ Shaded region is below x-axis area = $32\ln 2-\frac{33}{2}$
	Applies integration by parts correctly to $(2x-8)\ln x$ to obtain either $(x^2-8x)\,ln\,x-\int x-8\ \mathrm{d}x$OE or $\frac{1}{4}(2x-8)^2\ln x-\int x-8+\frac{16}{x}\,\mathrm{d}x$ Condone missing brackets or omission of dx	1.1a	M1	
	Completes integration fully to obtain either $(x^2-8x)\ln x-\frac{x^2}{2}+8x$ OE or $\frac{1}{4}(2x-8)^2\ln x-\frac{x^2}{2}+8x-16\ln x$ OE	1.1b	A1	
	Substitutes limits 1 and 4 into their integrated function and subtracts either way round	1.1a	M1	
	Completes reasoned argument to correctly obtain $\frac{33}{2}-32\ln 2$ or $32\ln 2-\frac{33}{2}$ AG Brackets must be correct throughout	2.1	R1	
	Explains change of sign due to shaded region **being below x-axis** This could be at an earlier stage eg swap limits explained but must still refer to the shaded region **being below x-axis**	2.4	E1	
	Subtotal		**6**	

	Question 14 Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)(i)	States $(\sin\theta)^{-1}$ or $\frac{1}{\sin\theta}$ $\sin^{-1}\theta$ scores B0 Ignore $\sin^{-1}\theta$ if a correct expression has already been written	1.2	B1	$\frac{1}{\sin\theta}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)(ii)	Uses chain rule or quotient rule to obtain $\pm k(\sin\theta)^{-2}\cos\theta$ OE or Multiplies and uses product rule and implicit differentiation to obtain $\pm k(\sin\theta)^{-2}\cos\theta$ This mark can be awarded for using $\frac{1}{\cos\theta}$ and differentiating to obtain $\pm k(\cos\theta)^{-2}\sin\theta$ OE Ignore wrong or missing angles	3.1a	M1	
	Obtains $-(\sin\theta)^{-2}\cos\theta$ OE	1.1b	A1	$\frac{dy}{d\theta} = -(\sin\theta)^{-2}\cos\theta$ $= -\frac{\cos\theta}{\sin\theta} \times \frac{1}{\sin\theta}$ $= -\operatorname{cosec}\theta\cot\theta$
	Completes **rigorous** argument to show the given result. Must either see separated fractions before final line or sight of $-\frac{\cot\theta}{\sin\theta}$ or $-\frac{1}{\tan\theta\sin\theta}$ or $-\frac{\cos\theta}{\sin\theta} \times \operatorname{cosec}\theta$ or Makes clear use of stated identities as part of the solution At some point the solution must have included $\frac{dy}{d\theta} =$ AG Condone change of order of functions at the end	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)(iii)	Substitutes $y = \operatorname{cosec}\theta$ OE or Draws a right angled triangle labelling hypotenuse as y and opposite as 1 PI by obtaining y^2 or $\frac{1}{y^2}$ in terms of $\cos\theta$	1.1b	B1	$\frac{\sqrt{\operatorname{cosec}^2\theta - 1}}{\operatorname{cosec}\theta}$
	Uses $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$ OE or Uses Pythagoras theorem to find missing adjacent side in the right angled triangle or Obtains $\cos^2\theta$ in terms of y	1.1a	M1	$= \frac{\sqrt{\cot^2\theta}}{\operatorname{cosec}\theta}$ $= \frac{\cot\theta}{\operatorname{cosec}\theta}$
	Completes **rigorous** argument to show the given result This must include **clear replacement** of $\operatorname{cosec}\theta$ and $\cot\theta$ **within the solution** using only sine and cosine functions to complete the argument prior to obtaining the answer given AG	2.1	R1	$= \frac{\cos\theta}{\sin\theta} \times \sin\theta$ $= \cos\theta$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)(i)	Obtains $\frac{\mathrm{d}x}{\mathrm{d}u} = -2\operatorname{cosec} u \cot u$ OE	1.1b	B1	$\frac{\mathrm{d}x}{\mathrm{d}u} = -2\operatorname{cosec} u \cot u$ $\mathrm{d}x = -2\operatorname{cosec} u \cot u \;\; \mathrm{d}u$ $= \int \frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u\sqrt{4\operatorname{cosec}^2 u - 4}}\mathrm{d}u$ $= \int \frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u\sqrt{4\cot^2 u}}\mathrm{d}u$ $= \int \frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u \times 2\cot u}\mathrm{d}u$ $= \int -\frac{1}{4\operatorname{cosec} u}du$ $= -\frac{1}{4}\int \sin u \;\mathrm{d}u$
	Makes complete substitution to obtain integrand of the form $\frac{P\operatorname{cosec} u \cot u}{Q\operatorname{cosec}^2 u\sqrt{R\operatorname{cosec}^2 u - 4}}$ OE or $\frac{P\operatorname{cosec} u \cot u}{Q\operatorname{cosec}^3 u\sqrt{1 - R\sin^2 u}}$ OE Ignore wrong or missing angles	1.1a	M1	
	Obtains correct integrand $\frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u\sqrt{4\operatorname{cosec}^2 u - 4}}$ or $\frac{-2\operatorname{cosec} u \cot u}{8\operatorname{cosec}^3 u\sqrt{1 - \sin^2 u}}$ OE	1.1b	A1	
	Uses appropriate Pythagorean-trig identity under the square root. Either $1 + \cot^2 u = \operatorname{cosec}^2 u$ or $1 - \sin^2 u = \cos^2 u$ Ignore wrong or missing angles	3.1a	M1	
	Obtains $k\int \sin u \;\mathrm{d}u$ with no errors seen in any trig identities Must have u and du	1.1b	A1F	
	Obtains $k = -\frac{1}{4}$ OE CSO	2.1	R1	
	Subtotal		**6**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)(ii)	Integrates $\int \sin u \; \mathrm{d}u$ to obtain $-\cos u$	1.1b	B1	$-\frac{1}{4}\int \sin u \; \mathrm{d}u = \frac{1}{4}\cos u + c$ $= \frac{1}{4}\frac{\sqrt{\left(\frac{x}{2}\right)^2 - 1}}{\left(\frac{x}{2}\right)} + c$ $= \frac{\sqrt{\frac{x^2-4}{4}}}{2x} + c$ $= \frac{\sqrt{x^2-4}}{4x} + c$
	Deduces $\cos u = \frac{\sqrt{\left(\frac{x}{2}\right)^2 - 1}}{\left(\frac{x}{2}\right)}$ OE	2.2a	M1	
	Completes **reasoned** argument to show given result Must have $+c$ throughout Validation by starting with $\frac{\sqrt{x^2-4}}{4x}$ and replacing x with $2\operatorname{cosec} u$ to achieve $\frac{1}{4}\cos u$ scores a maximum of B1M1R0	2.1	R1	
	Subtotal		**3**	

	Question 15 Total		**16**	

	Question Paper Total		**100**	