A-level

FURTHER MATHEMATICS 7367/3D

Paper 3 Discrete

Mark scheme

June 2021

Version: 1.0 Final Mark Scheme

216A7367/3D/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Ticks correct answer	1.1b	B1	The first activity in a critical path has an earliest start time of zero
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	56
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(a)(i)	Sets up a model of finding a maximum spanning tree by listing at least 4 correct labelled arcs	3.3	M1	Using Prim's algorithm: *X*–*B*: 3.0 *B*–*C*: 3.5 *B*–*E*: 3.0 *E*–*G*: 4.5 *G*–*H*: 5.5 *D*–*E*: 4.0 *A*–*D*: 3.0 *C*–*F*: 2.5
	Uses their model to find the correct 8 arcs of the maximum spanning tree	3.4	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
3(a)(ii)	Finds the total of the weights of the 8 arcs from their model	3.1b	M1	3.0 + 3.5 + 3.0 + 4.5 + 5.5 + 4.0 + 3.0 + 2.5 = 29 tonnes
	Finds the correct estimate for the maximum amount of precious metal with units from their model	3.2a	A1F	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
3(b)	Explains that the values used in calculating **(a)(ii)** are estimates	2.4	E1	The values used to arrive at 29 tonnes are estimates, so 29 tonnes is an estimate for the maximum amount of precious metal The true value may be more than or less than 29 tonnes
	Infers that the true maximum value may be more or less than the estimate	2.2b	E1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
3(c)	Evaluates the removal of *CF* from their model	3.5a	M1	*CF* is part of the spanning tree that maximises the estimate for the amount of precious metal that can be excavated. Either *EF* or *HF* will need excavating instead as they both have a weight of 3.0 This will reduce the estimate for the maximum amount of precious metal by 0.5 tonnes to 28.5 tonnes.
	Determines that the estimate for the maximum amount of precious metal will decrease by 0.5 tonnes to 28.5 tonnes CAO Condone omission of units	1.1b	A1	
	Total		**2**	

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Finds the correct Hamiltonian cycle starting at the depot using the nearest neighbour algorithm Condone omission of *B*–*O*	3.1a	M1	*O*–*F*–*D*–*E*–*C*–*A*–*B*–*O* (10 + 6 + 6 + 13 + 7 + 13 + 12) = 67 miles Time taken to drive this distance = (67 / 40) × 60 = 100.5 minutes Time taken to complete all 6 biofuel deliveries = 6 × 30 = 180 minutes Therefore an upper bound for T is 100.5 + 180 = 280.5
	Finds the correct total distance PI	1.1b	A1	
	Finds the correct total time, in minutes or hours, using their total distance	1.1b	A1F	
	Finds the time taken, in minutes or hours, to complete all 6 biofuel deliveries PI	1.1a	M1	
	Finds correctly the upper bound on T CAO	3.2a	A1	
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Explains that *CE* was part of the Hamiltonian cycle and that *A* is now the nearest neighbour to *E*, so *AE* should be used instead	2.4	M1	As *CE* cannot be used, *AE* and then *AC* must be used instead. This makes a new Hamiltonian cycle of *O*–*F*–*D*–*E*–*A*–*C*–*B*–*O* (10 + 6 + 6 + 14 + 7 + 8 + 12) = 63 miles This reduces the upper bound on T by 6 to 274.5
	Deduces correctly that the upper bound on T reduces by 6 or reduces to 274.5	2.2a	A1	
	Total		**2**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	States correctly at least three of the names of the group conditions or Describes correctly at least one group condition	1.1b	B1	For a set of elements to form a group under a binary operation: **Closure** For all elements a & b in S, $a*b$ is also in S **Identity** There exists an element e in S such that $e*a=a=a*e$ for all elements a in S **Inverse** For all elements a in S, there exists an element b in S such that $a*b=e=b*a$ **Associativity** For any three elements a, b & c in S, $(a*b)*c=a*(b*c)$
	Describes correctly at least two group conditions	1.1b	B1	
	Describes correctly all four group conditions and no others	1.1b	B1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)(i)	Explains correctly that G is an abelian group because multiplication modulo n is a commutative binary operation	2.4	E1	G is abelian as multiplication modulo n is a commutative binary operation
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)(ii)	Finds and simplifies 10^2 and 10^3 modulo 13 (condone lack of modulo notation)	1.1a	M1	$10^1 \equiv 10 \pmod{13}$ $10^2 \equiv 9 \pmod{13}$ $10^3 \equiv 12 \pmod{13}$ $10^4 \equiv 3 \pmod{13}$ $10^5 \equiv 4 \pmod{13}$ $10^6 \equiv 1 \pmod{13}$ Hence, the order of G is 6
	Finds correctly the order of G	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(c)	States that 1 is the identity element of G	1.1b	B1	1 is the identity of G To show this, let g be an element of G. Then $1 \times_{13} g = g$ and $g \times_{13} 1 = g$ Hence, as both products return the element g, then 1 is the identity of G
	Reasons that any element of G multiplied by 1 remains unchanged (either left-multiplication or right-multiplication) or Shows that each element of G when multiplied by 1 (either left-multiplication or right-multiplication) remains unchanged	1.1a	M1	
	Completes a rigorous mathematical proof, including an argument based upon the commutativity of multiplication modulo 13, the abelian property of G or both left- and right-multiplication, and a concluding statement	2.1	R1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
5(d)	Finds at least one proper, non-trivial subgroup of G Condone poor notation	1.1b	B1	$(\langle 9 \rangle, \times_{13})$ $(\langle 12 \rangle, \times_{13})$
	Finds both proper, non-trivial subgroups of G and no other subgroups.	1.1b	B1	
	Total		**2**	

	Question total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Uses Euler's formula for connected planar graphs	3.1a	M1	$(x+1)^2-(25+2x-2y)$ $+(y-1)^2=2$
	Finds the correct equation in terms of x and y	1.1b	A1	$x^2+2x+1-25-2x+2y$ $+y^2-2y+1=2$
	Expands and simplifies their equation	1.1a	M1	$x^2+y^2=25$
	Solves the equation to find the two correct pairs of integer values for x and y	1.1b	A1	$x=3,\ y=4$ or $x=4,\ y=3$
	Translates the correct values of x and y into one correct set of values for the number of vertices, edges and faces	3.2a	A1	For $x=3,\ y=4$ vertices $=16$ edges $=23$ faces $=9$
	Finds both correct sets of values for the number of vertices, edges and faces and not others	3.2a	A1	For $x=4,\ y=3$ vertices $=25$ edges $=27$ faces $=4$
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Explains that K_6 contains a subgraph which is K_5 or Explains that K_6 contains a subgraph which is $K_{3,3}$	2.4	E1	K_6 has a subgraph which is K_5 Kuratowski's theorem states that a graph which contains a subgraph that is a subdivision of K_5 or $K_{3,3}$ is not planar
	Completes a rigorous mathematical proof, including reference to Kuratowski's theorem	2.1	R1	Hence, K_6 is not planar
	Total		**2**	

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Translates the information in the pay-off matrix into at least one correct expression involving the three probability variables	3.1a	M1	There are no dominated strategies $3p_1 + 2p_2 + 6p_3 \geq v$ $p_1 + 3p_2 + 2p_3 \geq v$ $4p_1 + 2p_2 + p_3 \geq v$ $p_1 + p_2 + p_3 \leq 1$ Maximise $P = v$ Subject to $3p_1 + 2p_2 + 6p_3 \geq v$ $p_1 + 3p_2 + 2p_3 \geq v$ $4p_1 + 2p_2 + p_3 \geq v$ $p_1 + p_2 + p_3 \leq 1$ $p_1, p_2, p_3 \geq 0$
	Finds all three correct inequalities involving the three probability variables and the value of the game	1.1b	A1	
	Writes down the correct condition for the sum of the probability variables	1.1b	B1	
	Formulates the situation as a linear programming problem, including use of the words ‘maximise’ and ‘subject to’, and the inclusion of the non-negativity conditions	1.1b	A1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)(i)	Translates their linear programming problem by introducing four slack variables in the column headings	3.1a	M1	
	Finds two correct rows	1.1b	A1	
	Finds all correct rows	1.1b	A1	

P	v	p_1	p_2	p_3	s_1	s_2	s_3	s_4	**value**
1	–1	0	0	0	0	0	0	0	0
0	1	–3	–2	–6	1	0	0	0	0
0	1	–1	–3	–2	0	1	0	0	0
0	1	–4	–2	–1	0	0	1	0	0
0	0	1	1	1	0	0	0	1	1

	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)(ii)	Uses the simplex algorithm to modify at least one non-pivot row correctly	3.1a	M1	
	Uses the simplex algorithm to find at least three rows correctly	1.1b	A1	
	Uses the simplex algorithm to find all rows correctly	1.1b	A1	
	Total		**3**	

P	v	p_1	p_2	p_3	s_1	s_2	s_3	s_4	**value**
1	0	–3	–2	–6	1	0	0	0	0
0	1	–3	–2	–6	1	0	0	0	0
0	0	2	–1	4	–1	1	0	0	0
0	0	–1	0	5	–1	0	1	0	0
0	0	1	1	1	0	0	0	1	1

Q	Marking instructions	AO	Marks	Typical solution
7(c)(i)	Interprets the final simplex tableau to find the optimal mixed strategy for Avon	3.2a	B1	The optimal mixed strategy for Avon is to play: strategy A_1 with probability 0.25, strategy A_2 with probability 0.70, and strategy A_3 with probability 0.05
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)(ii)	Deduces the value of the game for Avon	2.2a	B1	The value of the game for Avon is $3 \times 0.25 + 2 \times 0.70 + 6 \times 0.05 = 2.45$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(d)	Explains correctly that Avon should play $\mathbf{A_2}$ each time	2.4	E1	To improve their outcome, Avon should play $\mathbf{A_2}$ each time Under these conditions, Avon's maximum expected pay-off will be 3
	Determines correctly that Avon's maximum expected pay-off will be 3	3.2a	E1	
	Total		**2**	

	Question total		**14**	

	Paper total		**50**	