AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

A-level
FURTHER MATHEMATICS
Paper 1

Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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4	
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7	
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10	
11	
12	
TOTAL	

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Answer **all** questions in the spaces provided.

1 The displacement of a particle from its equilibrium position is x metres at time t seconds.

The motion of the particle obeys the differential equation

$$\frac{d^2x}{dt^2} = -9x$$

Calculate the period of its motion in seconds.

Circle your answer.

[1 mark]

$\frac{\pi}{9}$ $\frac{2\pi}{9}$ $\frac{\pi}{3}$ $\frac{2\pi}{3}$

Do not write outside the box

2 Simplify

$$\frac{\cos\left(\frac{6\pi}{13}\right) + \mathrm{i}\sin\left(\frac{6\pi}{13}\right)}{\cos\left(\frac{2\pi}{13}\right) - \mathrm{i}\sin\left(\frac{2\pi}{13}\right)}$$

Tick (✓) **one** box.

[1 mark]

$\cos\left(\frac{8\pi}{13}\right) + \mathrm{i}\sin\left(\frac{8\pi}{13}\right)$ ☐

$\cos\left(\frac{8\pi}{13}\right) - \mathrm{i}\sin\left(\frac{8\pi}{13}\right)$ ☐

$\cos\left(\frac{4\pi}{13}\right) + \mathrm{i}\sin\left(\frac{4\pi}{13}\right)$ ☐

$\cos\left(\frac{4\pi}{13}\right) - \mathrm{i}\sin\left(\frac{4\pi}{13}\right)$ ☐

Turn over for the next question

3 Given that $y = \operatorname{sech} x$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$

Tick (✓) **one** box.

[1 mark]

$\operatorname{sech} x \tanh x$ ☐

$-\operatorname{sech} x \tanh x$ ☐

$\operatorname{cosech} x \coth x$ ☐

$-\operatorname{cosech} x \coth x$ ☐

4 The vector **v** is an eigenvector of the matrix **N** with corresponding eigenvalue 4

The vector **v** is also an eigenvector of the matrix **M** with corresponding eigenvalue 3

Given that

$$\mathbf{N}\mathbf{M}^2\mathbf{v} = \lambda\mathbf{v}$$

find the value of λ

Circle your answer.

[1 mark]

10 24 36 144

Do not write outside the box

5 It is given that $z = -\frac{3}{2} + \mathrm{i}\frac{\sqrt{11}}{2}$ is a root of the equation

$$z^4 - 3z^3 - 5z^2 + kz + 40 = 0$$

where k is a real number.

5 (a) Find the other three roots.

[5 marks]

5 (b) Given that $x \in \mathbb{R}$, solve

$$x^4 - 3x^3 - 5x^2 + kx + 40 < 0$$

[1 mark]

Turn over ▶

Do not write outside the box

6 (a) Given that $|x| < 1$, prove that

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

[4 marks]

Do not write outside the box

6 (b) Solve the equation

$$20\,\text{sech}^2 x - 11\tanh x = 16$$

Give your answer in logarithmic form.

[4 marks]

Do not write outside the box

7 The matrix **M** is defined as

$$\mathbf{M} = \begin{bmatrix} 1 & 7 & -3 \\ 3 & 6 & k+1 \\ 1 & 3 & 2 \end{bmatrix}$$

where k is a constant.

7 (a) (i) Given that **M** is a non-singular matrix, find $\mathbf{M}^{-1}$ in terms of k

[5 marks]

Do not write outside the box

7 (a) (ii) State any restrictions on the value of k

[1 mark]

7 (b) Using your answer to part **(a)(i)**, solve

$$x + 7y - 3z = 6$$
$$3x + 6y + 6z = 3$$
$$x + 3y + 2z = 1$$

[3 marks]

Do not write outside the box

8 (a) The complex number w is such that

$$\arg(w + 2\mathrm{i}) = \tan^{-1}\frac{1}{2}$$

It is given that $w = x + \mathrm{i}y$, where x and y are real and $x > 0$

Find an equation for y in terms of x

[2 marks]

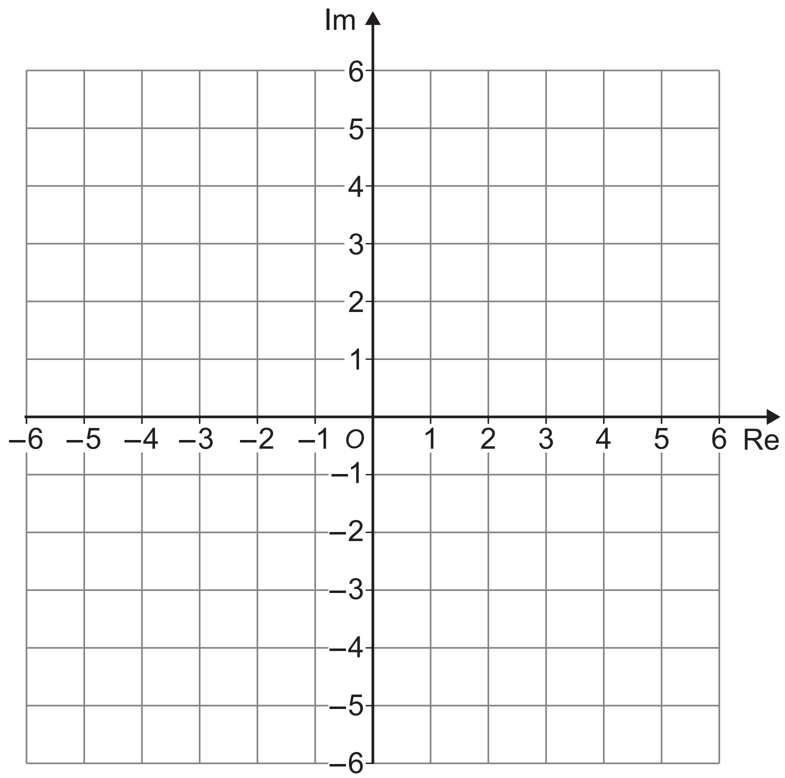
8 (b) The complex number z satisfies both

$$-\frac{\pi}{2} \leq \arg(z + 2\mathrm{i}) \leq \tan^{-1}\frac{1}{2} \qquad \textbf{and} \qquad |z - 2 + 3\mathrm{i}| \leq 2$$

The region *R* is the locus of z

Sketch the region *R* on the Argand diagram below.

[4 marks]



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8 (c) z_1 is the point in R at which $|z|$ is minimum.

8 (c) (i) Calculate the exact value of $|z_1|$

[3 marks]

8 (c) (ii) Express z_1 in the form $a + \mathrm{i}b$, where a and b are real.

[2 marks]

9 Roberto is solving this mathematics problem:

> The curve C_1 has polar equation
>
> $$r^2 = 9\sin 2\theta$$
>
> for all possible values of θ
>
> Find the area enclosed by C_1

Roberto's solution is as follows:

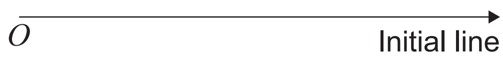
$$A = \frac{1}{2}\int_{-\pi}^{\pi} 9\sin 2\theta \ \mathrm{d}\theta$$

$$= \left[-\frac{9}{4}\cos 2\theta\right]_{-\pi}^{\pi}$$

$$= 0$$

9 (a) Sketch the curve C_1

[2 marks]



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9 (b) Explain what Roberto has done wrong.

[2 marks]

9 (c) Find the area enclosed by C_1

[2 marks]

9 (d) P and Q are distinct points on C_1 for which r is a maximum.
P is above the initial line.

Find the polar coordinates of P and Q

[2 marks]

Turn over ▶

Do not write outside the box

9 (e) The matrix $\mathbf{M} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ represents the transformation T

T maps C_1 onto a curve C_2

9 (e) (i) T maps P onto the point P'

Find the polar coordinates of P'

[4 marks]

Do not write outside the box

9 (e) (ii) Find the area enclosed by C_2

Fully justify your answer.

[2 marks]

Turn over for the next question

10 In this question all measurements are in centimetres.

A small, thin laser pen is set up with one end at $A(7, 2, -3)$ and the other end at $B(9, -3, -2)$

A laser beam travels from A to B and continues in a straight line towards a large thin sheet of glass.

The sheet of glass lies within a plane Π_1 which is modelled by the equation

$$4x + py + 5z = 9$$

where p is an integer.

10 (a) The laser beam hits Π_1 at an acute angle α, where $\sin\alpha = \dfrac{\sqrt{15}}{75}$

Find the value of p

[6 marks]

Do not write outside the box

10 (b) A second large sheet of glass lies on the other side of Π_1

This second sheet lies within a plane Π_2 which is modelled by the equation

$$4x + py + 5z = -5$$

Calculate the distance between the sheets of glass.

[2 marks]

10 (c) The point $A(7, 2, -3)$ is reflected in Π_1

Find the coordinates of the image of A after reflection in Π_1

[4 marks]

Turn over ▶

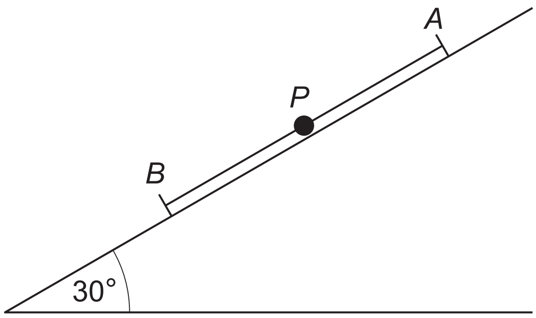
Do not write outside the box

11 **In this question use g as $10\,\text{m}\,\text{s}^{-2}$**

A smooth plane is inclined at 30° to the horizontal.
The fixed points *A* and *B* are 3.6 metres apart on the line of greatest slope of the plane, with *A* higher than *B*

A particle *P* of mass 0.32 kg is attached to one end of each of two light elastic strings. The other ends of these strings are attached to the points *A* and *B* respectively.

The particle *P* moves on a straight line that passes through *A* and *B*



The natural length of the string *AP* is 1.4 metres.
When the extension of the string *AP* is e_A metres, the tension in the string *AP* is $7e_A$ newtons.
The natural length of the string *BP* is 1 metre.
When the extension of the string *BP* is e_B metres, the tension in the string *BP* is $9e_B$ newtons.

The particle *P* is held at the point between *A* and *B* which is 0.2 metres from its equilibrium position and lower than its equilibrium position.
The particle *P* is then released from rest.

At time t seconds after *P* is released, its displacement towards *B* from its equilibrium position is x metres.

11 (a) Show that during the subsequent motion the object satisfies the equation

$$\ddot{x} + 50x = 0$$

Fully justify your answer.

[5 marks]

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11 (b) The experiment is repeated in a large tank of oil.
During the motion the oil causes a resistive force of kv newtons to act on the particle, where $v\,\mathrm{m\,s^{-1}}$ is the speed of the particle.

The oil causes critical damping to occur.

11 (b) (i) Show that $k = \frac{16\sqrt{2}}{5}$

[3 marks]

Turn over ▶

Do not write outside the box

11 (b) (ii) Find x in terms of t, giving your answer in exact form.

[6 marks]

11 (b) (iii) Calculate the maximum speed of the particle.

[5 marks]

Turn over for the next question

Do not write outside the box

12 The Argand diagram shows the solutions to the equation $z^5 = 1$

12 (a) Solve the equation

$$z^5 = 1$$

giving your answers in the form $z = \cos\theta + \mathrm{i}\sin\theta$, where $0 \le \theta < 2\pi$

[2 marks]

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12 (b) Explain why the points on an Argand diagram which represent the solutions found in part **(a)** are the vertices of a regular pentagon.

[2 marks]

____

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____

12 (c) Show that if $c = \cos\theta$, where $z = \cos\theta + \mathrm{i}\sin\theta$ is a solution to the equation $z^5 = 1$, then c satisfies the equation

$$16c^5 - 20c^3 + 5c - 1 = 0$$

[5 marks]

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12 (d) The Argand diagram on page 22 is repeated below.

Explain, with reference to the Argand diagram, why the expression

$$16c^5 - 20c^3 + 5c - 1$$

has a repeated quadratic factor.

[3 marks]

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12 (e) O is the centre of a regular pentagon $ABCDE$ such that $OA = OB = OC = OD = OE = 1$ unit.
The distance from O to AB is h

By solving the equation $16c^5 - 20c^3 + 5c - 1 = 0$, show that

$$h = \frac{\sqrt{5}+1}{4}$$

[5 marks]

END OF QUESTIONS

There are no questions printed on this page

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ANSWER IN THE SPACES PROVIDED

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Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**

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