16+ Scholarship — Specimen Paper — Mathematics 1

1. (i) $8-12x$

   (ii) $2x^2+8x-5+3x = 2x^2+11x-5$

   (iii) $2x(2x-3)$

   (iv) $(3x+2)(x-4)$

2. (i) Using Pythagoras' Theorem, $DC = \sqrt{13^2-5^2} = 12$cm.

   (ii) Since $BC = 5$ cm, and M is half-way from A to C (and from B to D), the vertical height of M above the base AB (of the triangle ABM is 2.5 cm. Thus the area of ABM is $\frac{1}{2}\times 2.5\times 12 = 15$ cm^2

3. (i) $v = 5+(-4)\times 3 = 5-12 = 7$

   (ii) $9u = u+2a$, and hence $8u = 2a$, so that $a = 4u$.

   (iii) We manipulate to obtain

$$\begin{aligned}\frac{PV}{T} &= nR\\ PV &= nRT\\ \frac{PV}{nR} &= T\end{aligned}$$

4. (i) The sale price is £60×0.8, or £48.

   (ii) If the original price is £x, then $0.8x = 40$, and hence $x = 50$. The original price was £50.

5. (i) (a) $11, 8, 5, 2, \mathbf{-1}, \mathbf{-4}, \ldots$ — numbers decrease by 3 each time.

   (b) $-1, 0, 3, 8, \mathbf{15}, \mathbf{35}, \ldots$ — successive squares minus 1.

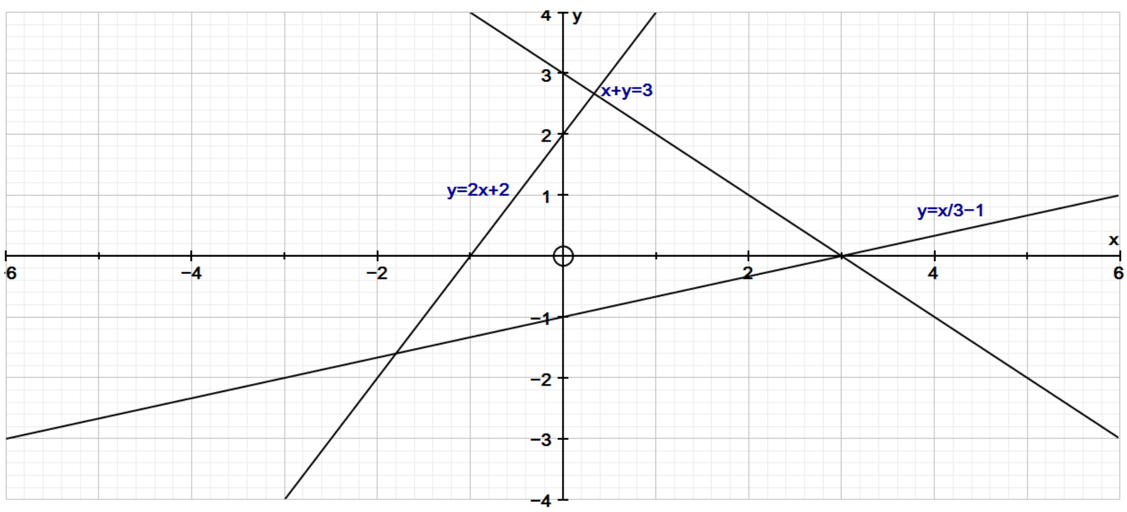
   (ii) $\frac{9}{8}$ and $\frac{11}{16}$.

   (iii) (a) $2n+1$,

   (b) $\frac{n^2}{n+1}$

6. The volume is $\pi\times 5^2\times 20 = 500\pi$ cm^3.

7. (i)



(ii) The lines $y = \frac{1}{3}x - 1$ and $x + y = 3$ meet at the point $(3,0)$.

(iii) Substituting, we obtain

$$\begin{aligned} \tfrac{1}{3}x - 1 &= 2x + 2 \\ -1 - 2 &= 2x - \tfrac{1}{3}x \\ \tfrac{5}{3}x &= -3 \end{aligned}$$

and hence $x = -\frac{9}{5}$, and hence $y = -\frac{8}{5}$.

8. (i) We have $3^{x+1} = \frac{1}{27} = 3^{-3}$, so $x + 1 = -3$, so $x = -4$.

(ii) We have $3 = 24x$, so $x = \frac{1}{8}$.

(iii) Solving,

$$\begin{aligned} \frac{2}{x+1} + 4 &= 9 \\ \frac{2}{x+1} &= 5 \\ 2 &= 5(x+1) \\ -3 &= 5x \end{aligned}$$

and hence $x = -\frac{3}{5}$.

9. (i) The enlargement has scale factor $\frac{80}{15} = \frac{16}{3}$. Thus the enlarged width is $12 \times \frac{16}{3} = 64$ cm.

(ii) This enlargement has scale factor $\frac{30}{45} = \frac{2}{3}$. Thus the area of the print is $2700 \times \left(\frac{2}{3}\right)^2 = 1200\ \text{cm}^2$.

10. (i) (a) $\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$.

(b) To score 17, I must choose 8 and 9 in some order. The probability of doing this is $\frac{1}{9} \times \frac{1}{9} + \frac{1}{9} \times \frac{1}{9} = \frac{2}{81}$.

(ii) Once the first face has been chosen, there are five faces, out of which we will choose the second face. Only one of these faces is opposite the first face. Thus the probability that the two faces are opposite each other is $\frac{1}{5}$.

11. (i) $3 \diamond (-3) = 3^2 + (-3)^2 = 18$.

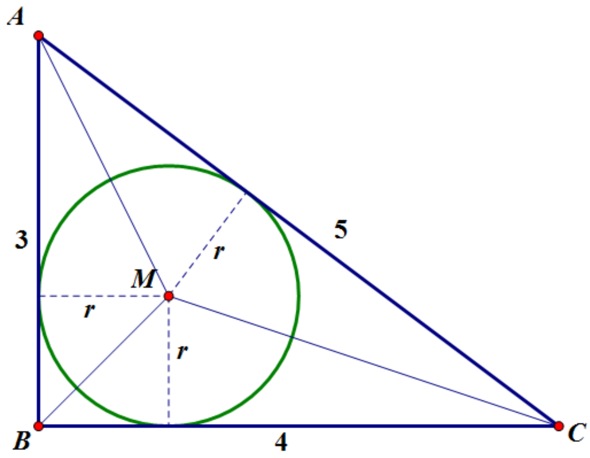
(ii) $2 \diamond (3 \diamond 4) = 2 \diamond (3^2 + 4^2) = 2 \diamond 25 = 2^2 + 25^2 = 629$.

(iii)

$$\begin{aligned} x \diamond x &= 3x + 9 \\ x^2 + x^2 &= 3x + 9 \\ 2x^2 - 3x - 9 &= 0 \\ (2x+3)(x-3) &= 0 \end{aligned}$$

and hence $x = -\frac{3}{2}, 3$.

12. Let M be the centre of the inscribed circle. The triangles ABM, BCM, CAM have bases $AB = 3$, $BC = 4$ and $CA = 5$, and heights r above their bases.



Thus these triangles have areas $\frac{3}{2}r$, $2r$ and $\frac{5}{2}r$ respectively. The sum of their areas is $6r$, which must equal the total area of the triangle ABC, which is $\frac{1}{2} \times 3 \times 4 = 6$. Thus $r = 1$, and so the area of the inscribed circle is $\pi r^2 = \pi$.