

ISEB

Independent Schools
Examinations Board

MATHS
ISEB Revision Guide
(3rd Edition)

Stephen Froggatt
and David Hanson

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Examinations Board

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GALORE PARK

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Details of other ISEB Revision Guides for Common Entrance, examination papers and Galore Park publications are available at www.galorepark.co.uk

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Introduction

This book has been written to help you revise. It is not a textbook, and the emphasis is on reminding rather than teaching you.

The book contains all the revision material you will need for your Common Entrance exam. In addition, Chapters 5 and 6 provide you with some practice in vital skills which will help you pass your exam.

Throughout you will find plenty of ‘Test yourself’ questions that will allow you to check that you have learned a section properly, and ‘Sample questions’ with which you practise the kind of questions that you will see in the exam. There are answers at the back of the book.

At the end of each chapter is a summary of what you should have learned. Make sure that you keep track of what you have and haven’t covered, and keep practising anything you’re unsure of.

Examinations are not designed to trip you up, but to give you a chance to demonstrate what you know and what you can do. Make sure that you have the facts at your fingertips so that you can show yourself off at your best.

Good luck!

The syllabus and your exam

The ISEB syllabus is revised regularly and is based on the programme of study for Key Stage 2 (11+) and Key Stage 3 (13+) of the National Curriculum. The syllabus expects that pupils will be familiar with the skills and knowledge of National Curriculum Key Stage 1 and the National Numeracy Strategy *Framework* for the early years.

If you are taking the ISEB Common Entrance 11+ exam, you will take one paper lasting one hour.

If you are taking the ISEB Common Entrance 13+ exam, you will take three papers:

- a Level 1, 2 or 3 calculator paper lasting one hour.
- a Level 1, 2 or 3 non-calculator paper lasting one hour.
- a mental arithmetic test lasting 30 minutes, which is the same for all levels.

A note for teachers

Sample questions and Test yourself questions have been modelled on ISEB Common Entrance examination questions and, therefore, frequently request the pupil to write on the paper. If you do not wish the pupil to mark this book, these questions are available as downloadable/CD products from Galore Park www.galorepark.co.uk

How to use this book

This book contains several symbols that will help you to revise only what you need to.

First, make sure you know which Level of the exam you are taking. If in doubt, ask your teacher.

- If you are taking Level 1, you do not need to revise anything tagged with the following symbols: ■■■□ ■■■■
- If you are taking Level 2, you will need to revise everything except the material tagged with the following symbol: ■■■■
- If you are taking Level 3, you will need to revise everything in this book.

Symbols such as this **5** refer to National Curriculum levels, a different system used in some schools. Your teacher will tell you if you need to refer to these.

S These symbols in the margin allow you to find the Summary and Test yourself sections quickly. Keep referring to these sections and check that you know what you need to.

TY

In the Summary sections, you may find some points with footnotes [*]. This is because you may be required to know slightly different things depending on which ISEB Level you are taking. Your teacher will explain this if necessary.

Sample questions have marks allocated, eg (2), as they would in the exam.

Tips on revising

Get the best out of your brain:

- Give your brain plenty of oxygen by exercising. You can revise effectively if you feel fit and well.
- Eat healthy food while you are revising. Your brain works better when you give it good fuel.
- Think positively. Give your brain positive messages so that it will want to study.
- Keep calm. If your brain is stressed it will not operate effectively.
- Take regular breaks during your study time.
- Get enough sleep. Your brain will carry on sorting out what you have revised while you sleep.

Get the most from your revision

- Don't work for hours without a break. Revise for 20–30 minutes and then take a five-minute break.

- Do good things in your breaks: listen to your favourite music, eat healthy food, drink some water, do some exercise or juggle. Don't read a book, watch TV or play on the computer; it will conflict with what your brain is trying to learn.
- When you go back to your revision review what you have just learnt.
- Regularly review the facts you have learnt.

Get motivated

- Set yourself some goals and promise yourself a treat when the exams are over.
- Make the most of all the expertise and talent available to you at school and at home. If you don't understand something, ask your teacher to explain.
- Get organised. Find a quiet place to revise and make sure you have all the equipment you need.
- Use yearly and weekly planners to help you organise your time so that you revise all subjects equally. (Available for download from www.galorepark.co.uk)

Know what to expect in the exam

- Use past papers to familiarise yourself with the format of the exam.
- Make sure you understand the language examiners use.

Before the exam

- Have all your equipment and pens ready the night before.
- Make sure you are at your best by getting a good night's sleep before the exam.
- Have a good breakfast in the morning.
- Take some water into the exam if you are allowed.
- Think positively and keep calm.

During the exam

- Have a watch on your desk. Work out how much time you need to allocate to each question and try to stick to it.
- Make sure you read and understand the instructions and rules on the front of the exam paper.
- Allow some time at the start to read and consider the questions carefully before writing anything.
- Read all the questions at least twice. Don't rush into answering before you have a chance to think about it.
- If you find a question is particularly hard, move on to the next one. Go back to it if you have time at the end.
- Check your answers make sense if you have time at the end.

Tips for the Maths exam

- Read the questions extra carefully. It is very easy to miss the word 'not' in a probability question, or the words 'as a percentage' in a number question.
- Show all your working. In a non-calculator question, failure to do this could result in loss of marks; in a calculator question it helps the examiner to check your method even if the final answer is wrong. Candidates often forget that marks can be awarded for working – it's not just about getting the right answer.
- Check that your answer sounds reasonable. A building 7 cm tall, or an average pupil mass of 987 kg, should make you think again. The most common way to get a crazy answer is to forget the units (centimetres and metres in the same question, for example).
- When drawing graphs and doing constructions, use a sharp pencil, and try to be as neat and precise as you can.

Useful resources

Study Skills by Elizabeth Holtom, ISBN: 9781902984599

Junior Maths Book 1 by David Hillard, ISEB: 9781905735211

Junior Maths Book 1 Answer Book by David Hillard, ISBN: 9781905735242

Junior Maths Book 2 by David Hillard, ISBN: 9781905735235

Junior Maths Book 2 Answer Book by David Hillard, ISBN: 9781905735242

Junior Maths Book 3 by David Hillard, ISBN: 9781905735266

Junior Maths Book 3 Answer Book by David Hillard, ISBN: 9781905735297

So you really want to learn Maths Book 1 by Serena Alexander, ISBN: 9781902984186

So you really want to learn Maths Book 1 Answer Book by Serena Alexander, ISBN:
9781902984193

So you really want to learn Maths Book 2 by Serena Alexander, ISBN: 9781902984315

So you really want to learn Maths Book 2 Answer Book by Serena Alexander, ISBN:
9781902984322

So you really want to learn Maths Book 3 by Serena Alexander, ISBN: 9781902984346

So you really want to learn Maths Book 3 Answer Book by Serena Alexander, ISBN:
9781902984353

Mathematics Pocket Notes by David E Hanson, ISBN: 9780903627146

Mathematics Questions at 11+ (Year 6) (Book A Questions) by David E Hanson,
ISBN: 9780903627092

Mathematics Questions at 11+ (Year 6) (Book A Answers) by David E Hanson,
ISBN: 9781907047152

Mathematics Questions at 11+ (Year 6) (Book B Questions) by David E Hanson,
ISBN: 9780903627351

Mathematics Questions at 11+ (Year 6) (Book B Answers) by David E Hanson,
ISBN: 9781907047145

Mixed Maths Exercises (Year 6) by Andrew Jeffrey, ISBN: 9780903627030

Mixed Maths Exercises (Year 7) by Andrew Jeffrey, ISBN: 9780903627047

Mixed Maths Exercises (Year 8 Lower) by Andrew Jeffrey, ISBN: 9780903627054

Mixed Maths Exercises (Year 8 Upper) by Andrew Jeffrey, ISBN: 9780903627078

Mixed Maths Exercises Answers by Andrew Jeffrey, (downloadable from
www.galorepark.co.uk)

All available from Galore Park: www.galorepark.co.uk

Chapter 1: 11+ Number

1.1 Properties of numbers

Types of number

- You should be familiar with some of the more common words used to describe numbers and their properties.

For this section it really helps if you know your tables well! Let's see why they are so important.

The multiplication table

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

You need to know your tables in several ways. First of all, learn them in columns so that you can list, for example, all multiples of seven. Then, be able to jump in at any point, and know that, e.g. $4 \times 8 = 32$, without thinking. Finally, you should be able to use it backwards and be able to give all factor pairs of a number (e.g. which numbers multiply to give thirty-six?).

Multiples

Multiples of 3 include 3, 6, 9, 12, 15, 18,

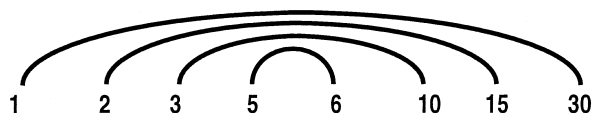
Multiples of 6 include 6, 12, 18, 24, 30, 36,

Think of multiples as the **results** in the times table of a number.

Factors

Factors of 30 are all the numbers which divide exactly into 30

We can make a factor rainbow.



Notice how they come in pairs.

$$30 = 1 \times 30 \quad \text{or} \quad 2 \times 15 \quad \text{or} \quad 3 \times 10 \quad \text{or} \quad 5 \times 6$$

Think of factors as the **questions** from the times table.

Primes

Quite simply, a prime is a number which has only two factors: 1 and itself.

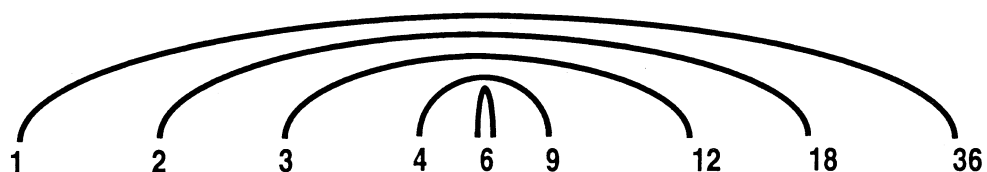
The first 100 prime numbers

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199	211	223	227	229
233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349
353	359	367	373	379	383	389	397	401	409
419	421	431	433	439	443	449	457	461	463
467	479	487	491	499	503	509	521	523	541

Obviously you do not need to learn all these! Try to be familiar with as many as you can, though. At the very least learn the first ten or so, and reach the point where you can recognise all numbers under 100 as either prime or not. Note that 1 is not prime.

Squares, cubes and roots

Any number multiplied by itself gives a **square**. In the factor rainbow this number joins to itself in the middle. Let's try this for the number 36



Notice how the 6 is joined to itself because $36 = 6 \times 6$

Multiply any number (n) by itself and the result is a square number (n^2). Any number multiplied by its square gives a cube number (n^3). It is an extremely good idea to know the squares of at least the first ten numbers and at least the first five cubes. Here are the first fifteen of each:

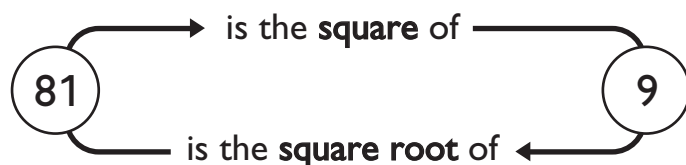
n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375

n^2 means n to the power of 2

2 is the index number

Similarly, for n^3 , 3 is the index number

Remember the connection between squares and square roots.



Cubes and cube roots work in a similar way.

Odd and even numbers and rules

There are several general rules about combining two numbers, depending on whether they are odd or even. You may wish to think about why these rules always work as they do.

odd + odd = even
 odd + even = odd
 even + odd = odd
 even + even = even

odd - odd = even
 odd - even = odd
 even - odd = odd
 even - even = even

odd × odd = odd
 odd × even = even
 even × odd = even
 even × even = even

There are no simple rules for division.
 Any division could be a fraction which is neither odd nor even.

Negative numbers

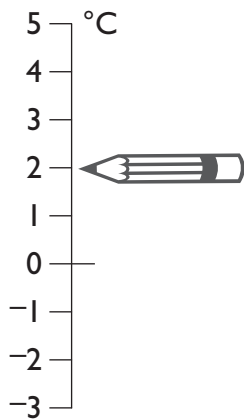
- You need to be able to add and subtract negative numbers in context, and sort them into order of size.

Real-life negatives

Where do we find negative numbers in everyday life? Temperatures are perhaps the most common, but we also talk about negative or overdrawn bank balances (in the red), as well as distances below sea level.

Brr! Below zero

Negative temperatures mean ice and snow. -3°C is 3 degrees below zero and so on. Let's extend the number line downwards to see what's going on.



Put your pencil on 2°C (as shown).

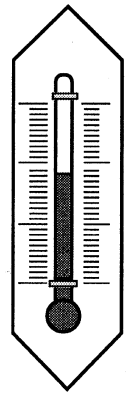
Now move down 3 degrees and you should end up with your pencil on -1°C

We write: $2 - 3 = -1$

Check also that: $5 - 8 = -3$

Adding takes us back up again.

$-2 + 6 = 4$



Adding -1 is equivalent to subtracting 1

$5 + -8$ is equivalent to $5 - 8 = -3$

Subtracting -1 is equivalent to adding 1

$10 - -4$ is equivalent to $10 + 4 = 14$

Putting negative numbers in order

Remember to think of the number line all the time. That way you will correctly say:

-15 is less than -8 which is less than 3 which is less than 11 and so on.

Sample questions

Try these sample questions for yourself. The answers are given at the back of the book.

Some of the questions involve ideas met in earlier work which may not be covered by the notes in this chapter. (See the Useful resources section in the Introduction if you need to refresh any skills or knowledge.)

1.1 (a) Which of these numbers are divisible by 3? (2)

15 26 36 45 56 114 1011

(b) Which of these numbers are multiples of 4? (2)

6 12 16 18 24 36 52

(c) Which of these numbers are factors of 24? (2)

2 3 4 6 8 12 24

1.2 Which of these numbers are prime? (2)

1 2 3 5 9 13 17

1.3 Here are some number cards.



From the cards, choose:

(a) a multiple of 6 (1)

(b) a factor of 6 (1)

(c) a square number (1)

(d) all the prime numbers (2)

(e) the number which is a common multiple of 4 and 6 (1)

(f) two numbers which have a product of 21 (1)

(g) three numbers which have a sum of 25, using each number only once. (2)

1.4 Write the next two numbers in these sequences.

(a) 15 20 25 30 35 ... (1)

(b) 57 54 51 48 45 ... (1)

(c) 17 13 9 5 1 ... (2)

1.5 Write the next two numbers in these sequences.

(a) 1 3 6 10 15 ... (2)

(b) 2 2 4 6 10 ... (2)

(c) 224 112 56 28 14 ... (2)

1.6 Gina is snorkelling.

(a) She dives to -2 m (2 metres below the water surface) and looks at a shark 3 metres below her. At what depth is the shark swimming? (1)

(b) On her next dive, Gina swims -2.3 metres and sees a jellyfish 1.4 metres above her. At what depth is the jellyfish swimming? (2)

1.7 (a) At 0700 one morning, the temperature inside the window was 4°C and the temperature outside the window was 11 degrees lower. What was the outside temperature? (1)

(b) By 0830 the temperature outside had risen to -3°C . By how many degrees had the outside temperature risen? (1)

1.2 Place value and ordering

Multiplying or dividing by 10 or 100

- You need to be able to multiply and divide whole numbers by 10 or 100 without a calculator.

Multiplying by 10 or 100

Multiplying by 10 or 100 is easy. It can be thought of as:

- writing zeros at the end of the *whole* numbers
- moving digits to the left so they have a higher place value
- moving the decimal point to the right.

Examples

H T U		H T U		H T U
2 3	×	1 0	=	2 3 0
1 7 4	×	1 0 0	=	1 7 4 0 0